CUTECat

Generating Testcases for Computational Laws through Concolic Execution

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Introduction

In this talk

- Program that encode laws
- What bugs these programs can have
- How Catala prevents some of them
- How I can detect them before they even happen

Computational law

- **Computational laws** encode algorithms: taxes, social benefits, etc.
- Administrations implement them as programs
- Critical: *e.g.* French military payroll system Louvois: 120k military personnel over- or under-paid, overpayments totalling 545M € to pay back

Structure of computational law: income tax example

Article 1

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Article 3 exception

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Article 4 exception

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Default logic

Transforming this law into a program

- **Input**: household description
  - income
  - number of children
- **Output**: computed income tax

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- Compute according to default logic
- Stay close to the text of the law

A simple Catala program

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# Article 1
The income tax is a fixed percentage of
the income.
```catala
scope IncomeTaxComputation:
 definition income_tax equals
   house.income * tax_rate
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# Article 2
The fixed percentage mentioned at
article 1 is 20%.
```catala
scope IncomeTaxComputation:
  definition tax_rate equals 20%
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~~~

- Literate programming

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# Article 3
If the income is less than $10,000, the
percentage mentioned at article 1 is 10%.
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scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.income <= $10,000
  consequence equals 10%
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# Article 4
For people in charge of 3 or more
children, the percentage mentioned at
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scope IncomeTaxComputation:
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- Literate programming
- Follows the exception/default structure of the law

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  - interpretation conflicts, *e.g.* `income = $9,000` and `children = 4`
  - unhandled cases
  - in Catala: ambiguity = crash
  - resolved by lawyers/administration if implementation is correct
- Other errors: division by zero, assertion error, etc.

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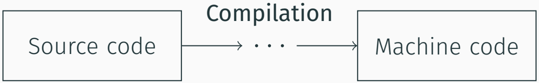
- Crashing in ambiguous situations
    - Catala doesn’t silently favor one interpretation
    - Happens when the code is executed
    - Risky when used in real life
- Anticipate those bugs when the code is written: we want to find bugs *a priori*

Background

Program compilation & execution

Source code

Program compilation & execution



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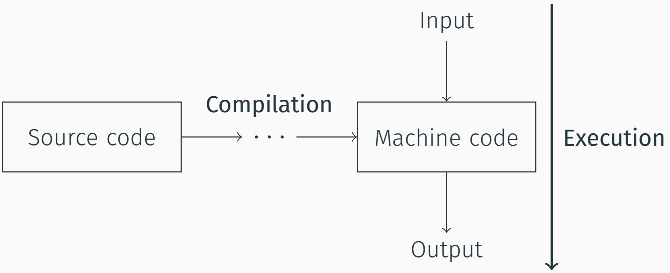

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Bugs are relative to the expected behavior: specification

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- … no guarantee that we can find every bug
- → Formal methods
    - rigorous reasoning about the program
    - find bugs systematically
    - guarantee formally that program has bugs/no bugs

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- Identify and prove properties
  - bugs: “The program always terminates without crashing”
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  - “Income tax increases with income”
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- Different properties are proven with different methods

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→ Concolic execution

Outline

Background

Concolic execution of default terms

Performance and usability improvements

Experimental evaluation

Concolic execution of default terms

Concolic execution: first example

Concolic = *conc*rete + symb*olic*

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if x > 0
    then 0
    else if y < 10
        then y
        else error
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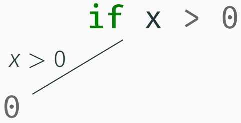

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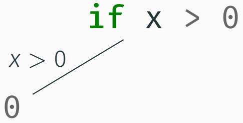

if x > 0

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Solver

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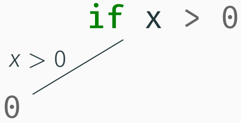

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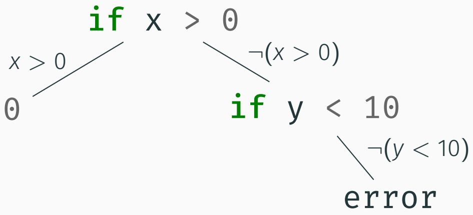

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2	0	20	error	$\neg(x > 0) \land \neg(y < 10)$	$\neg(x > 0) \land y < 10$

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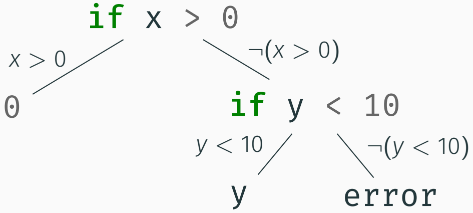

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3	0	9	9	$\neg(x > 0) \land y < 10$	-

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Default terms: syntax

Source code $\xrightarrow{\text{compiler}}$ **default terms**

Expressions $e ::= \langle e_1, \ldots, e_n \mid b_{default} :- e_{default} \rangle$

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$$\begin{array}{lcll} \textbf{Values} & v & ::= & \texttt{true} \mid \texttt{false} \mid n \mid \ldots \\ & & \mid & \varnothing \\ & & \mid & \circledast \end{array}$$

Default terms: semantics

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\begin{array}{l}
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\quad \langle \mid \texttt{income} \leq \$10,000 \ \texttt{:-}\ 10\% \rangle, \\
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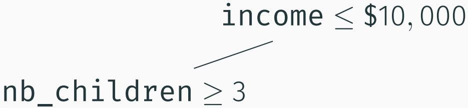

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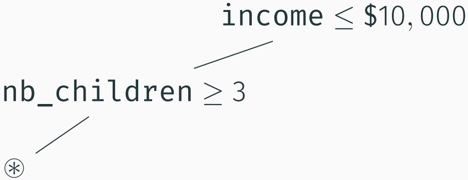

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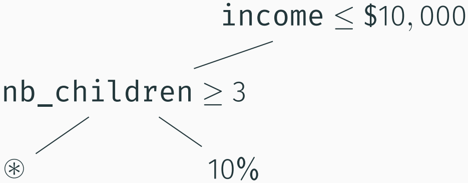

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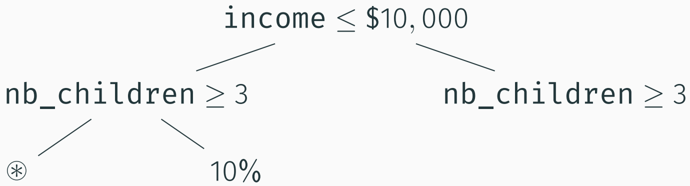
$\circledast$

10%

Concolic execution of default terms

$$\langle\langle \mid \texttt{income} \leq \$10,000 \texttt{ :- } 10\%\rangle, \langle \mid \texttt{nb\_children} \geq 3 \texttt{ :- } 15\%\rangle \mid \texttt{true} \texttt{ :- } 20\%\rangle$$

`income` = \$11,000; `nb_children` = 4



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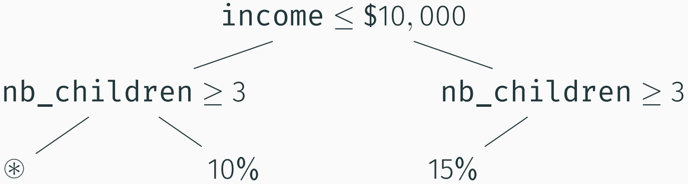
$\texttt{income} = \$11{,}000; \quad \texttt{nb\_children} = 4$



$\texttt{income} \leq \$10,000$

$\texttt{nb\_children} \geq 3$

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$\circledast$

10%

15%

Concolic execution of default terms

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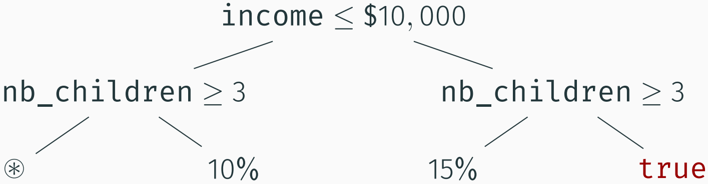
$\texttt{income} = \$11{,}000;\quad \texttt{nb\_children} = \texttt{2}$



$\texttt{income} \leq \$10,000$

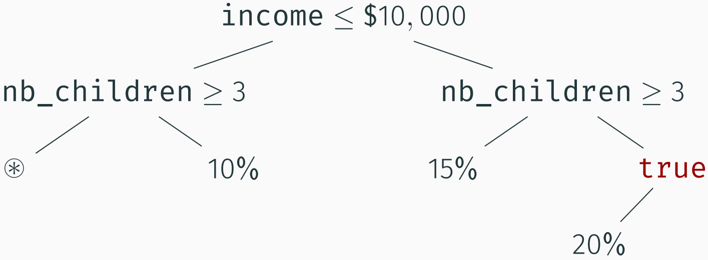
$\texttt{nb\_children} \geq 3$ $\qquad$ $\texttt{nb\_children} \geq 3$

$\circledast$ $\qquad$ 10% $\qquad$ 15% $\qquad$ $\texttt{true}$

Concolic execution of default terms

$\langle\langle\ |\ \texttt{income} \leq \$10,000\ \texttt{:-}\ 10\%\rangle, \langle\ |\ \texttt{nb\_children} \geq 3\ \texttt{:-}\ 15\%\rangle\ |\ \texttt{true}\ \texttt{:-}\ 20\%\rangle$

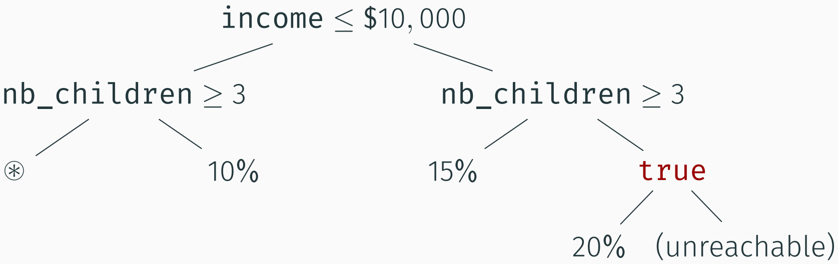
$\texttt{income} = \$11{,}000; \quad \texttt{nb\_children} = 2$



Concolic execution of default terms

$$\langle\langle \mid \texttt{income} \leq \$10,000 \;\texttt{:-}\; 10\%\rangle, \langle \mid \texttt{nb\_children} \geq 3 \;\texttt{:-}\; 15\%\rangle \mid \texttt{true} \;\texttt{:-}\; 20\%\rangle$$

$\texttt{income} = ???; \qquad \texttt{nb\_children} = ???$



Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.

~~~
# Article 3
If the income is less than $10,000, the percentage mentioned at article 1 is 10%.
```catala
scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.income <= $10,000
  consequence equals 10%
~~~

# Article 4
For people in charge of 3 or more children, the percentage mentioned at article 1
is 15%.
```catala
scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.nb_children >= 3
  consequence equals 15%
```
```
```

Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.

~~~
# Article 3
If the income is less than $10,000, the percentage mentioned at article 1 is 10%.
```catala
scope IncomeTaxComputation:
 exception children definition tax_rate
  under condition house.income <= $10,000
  consequence equals 10%
```

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For people in charge of 3 or more children, the percentage mentioned at article 1
is 15%.
```catala
scope IncomeTaxComputation:
 label children
 exception definition tax_rate
  under condition house.nb_children >= 3
  consequence equals 15%
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Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.

```
⟨
  ⟨ | income ≤ $10,000 :- 10%⟩,
  ⟨ | nb_children ≥ 3 :- 15%⟩
  | true :- 20%
⟩
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Fixing the interpretation conflict

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→ it becomes an exception to the exception.

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Performance and usability improvements

Performance optimizations using reordering

Independence of exception evaluation order

Performance optimizations using reordering

Theorem (Independence of exception evaluation order)

If there is a default value v such that

$$\langle \dots, e_i, \dots, e_j, \dots \mid b_{default} :- e_{default} \rangle \longrightarrow^* v,$$

then

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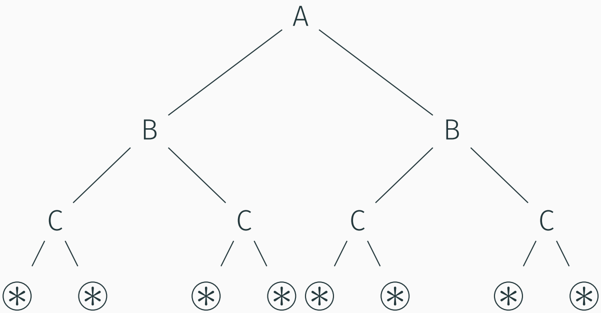
Example:

$$\langle A, B, C, \circledast \mid b_{default} :- e_{default} \rangle \sim \langle \circledast, A, B, C \mid b_{default} :- e_{default} \rangle$$

Performance optimizations using reordering – Example

$$\langle A\,,\ B\,,\ C\,,\ \circledast \ \mid\ b_{default}\ :-\ e_{default}\rangle \sim \langle \circledast\,,\ A\,,\ B\,,C \ \mid\ b_{default}\ :-\ e_{default}\rangle$$



Usability improvement

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Query: income > $10,000 ?

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- Difficult to compute by hand

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- Find more usable input values using **soft constraints**

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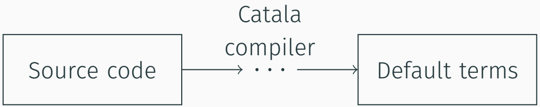
~~~~

Query: income > $10,000 ? → Answer: ~~$10,000.01~~ **$11,000**

- Difficult to compute by hand
- Find more usable input values using soft constraints
  - *e.g.* **round to $1,000**

Implementation of CUTECat



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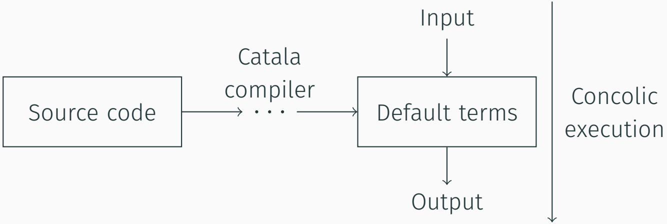

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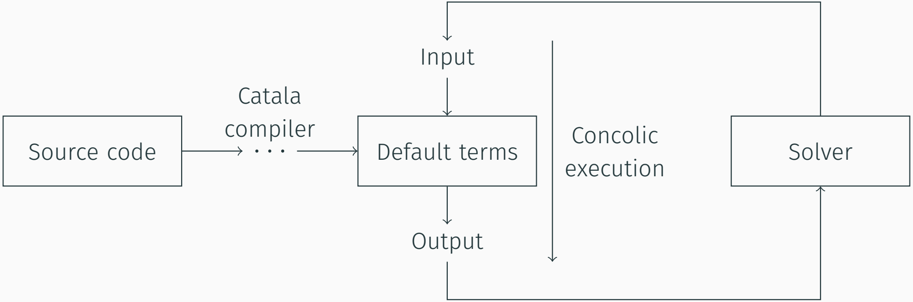

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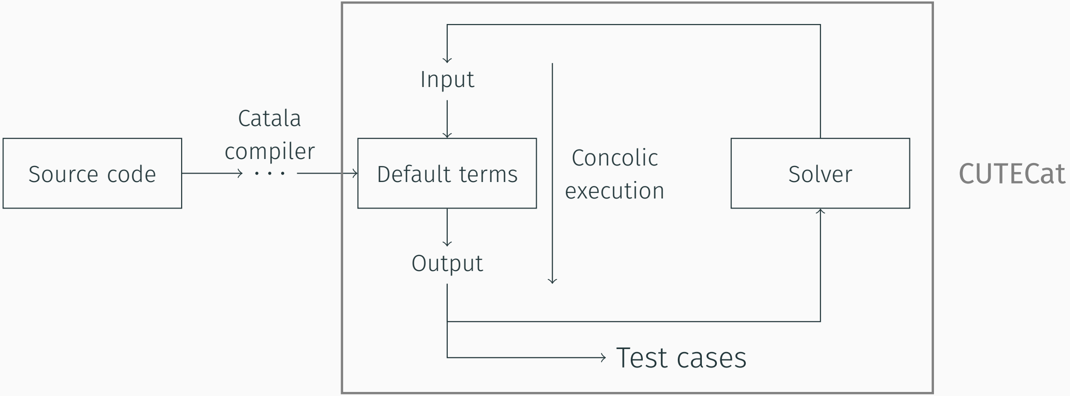

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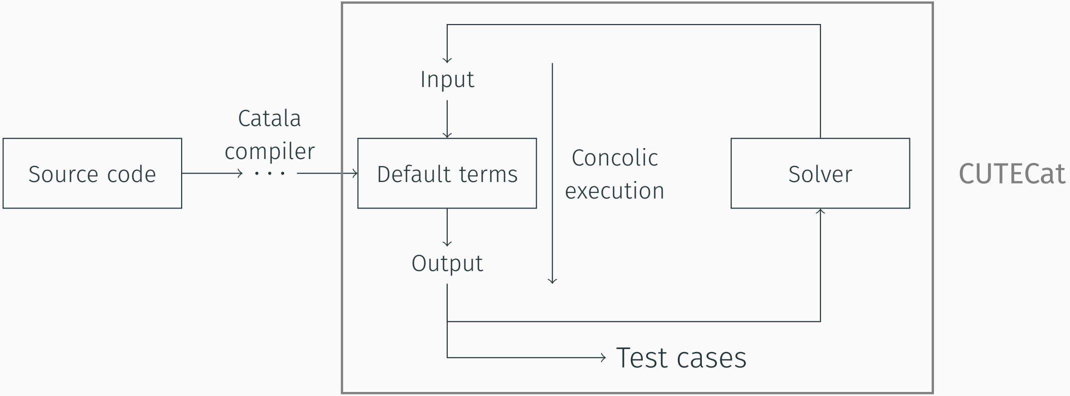

- 3.4k lines of OCaml code
- Z3 SMT Solver

Experimental evaluation

Code base

Law	Lines of law in Markdown	Lines of Catala	Total
French housing benefits	5736	8615	14351

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Law	Lines of law in Markdown	Lines of Catala	Total
French housing benefits	**5736**	**8615**	**14351**
US Tax code § 132	35	56	91
Minimum wage	74	161	235
Family quotient	36	165	201
Handwritten unit tests	139	699	838

Performance on small programs

	Time (s)			
Law	No optimizations	Incremental	All opt.	Generated tests
US Tax code	0.27	0.02	0.02	10
Minimum wage	1.01	0.08	0.08	17
Family quotient	82.61	5.21	4.34	381

Case study: housing benefits

Key results

- 186,390 test cases generated in **7h of CPU time**
- 99.83% of tests satisfy soft constraints
- Able to find a conflict
- 4.5x overhead w.r.t. concrete execution
- 366s spent in solver, the rest in evaluation

Conclusion

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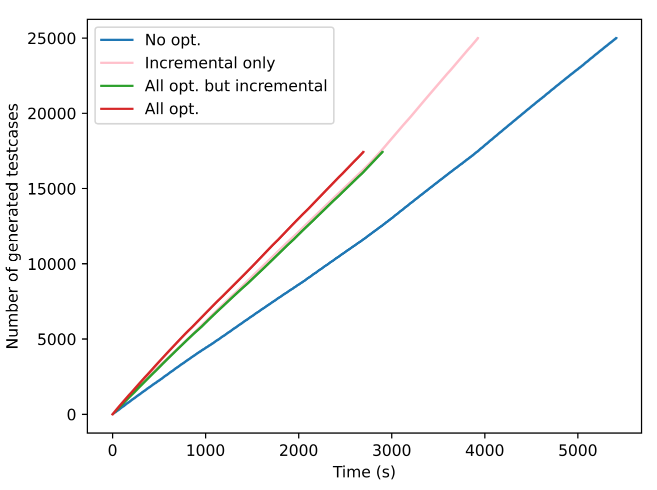
Contact, ESOP'25 preprint, slides: pierregoutagny.fr

References i

Ablation study



Generated tests vs time