# CUTECat

## Concolic Execution for Computational Law

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# Introduction

# Computational law

- **Computational laws** encode algorithms: taxes, social benefits, etc.
- Administrations implement them as programs
- Critical: *e.g.* French military payroll system Louvois: 120k military personnel over- or under-paid, overpayments totalling 545M € to pay back

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## Article 4

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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### Article 2 — default case

The fixed percentage mentioned at article 1 is 20%.

### Article 3 — exception

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### Article 4 — exception

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

Default logic

# The Catala domain-specific language

**# Article 1**
The income tax is a fixed percentage of the income.

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scope IncomeTaxComputation:
 definition income_tax equals
   house.income * tax_rate
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**# Article 2**
The fixed percentage mentioned at article 1 is 20%.

```catala
scope IncomeTaxComputation:
  definition tax_rate equals 20%
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- Literate programming

**# Article 3**
If the income is less than $10,000, the percentage mentioned at article 1 is 10%.

```catala
scope IncomeTaxComputation:
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  consequence equals 10%
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**# Article 4**
For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

```catala
scope IncomeTaxComputation:
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  under condition house.nb_children >= 3
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- Literate programming
- Follows the exception/default structure of the law

# Kinds of error

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  - interpretation conflicts, *e.g.* `income = $9,000` and `children = 4`
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  - in Catala: ambiguity = runtime error
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- Other errors: division by zero, assertion error, etc.

# Finding errors

What properties do we want in our search for errors?

- Find errors automatically
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→ Symbolic execution

- Avoid some common obstacles for free:
  - no loops or memory
  - all programs terminate
- But some features are hard to encode symbolically

# Outline

Concolic execution of default terms

Performance and usability improvements

Experimental evaluation

# Concolic execution of default terms

# Concolic execution: first example

Concolic = *conc*rete + symb*olic*

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if x > 0
    then 0
    else if y < 10
        then y
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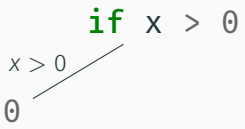


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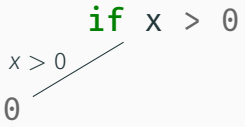


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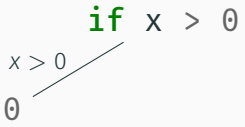


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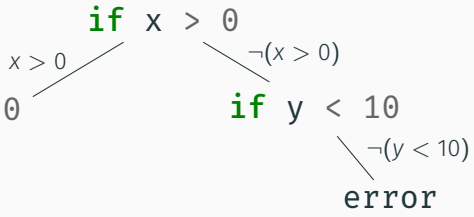


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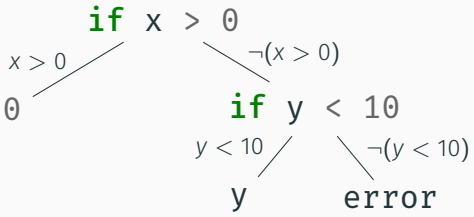


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| 3 | 0 | 9 | 9 | $\neg(x > 0) \wedge y < 10$ | - | |

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Source code $\xrightarrow{\text{compiler}}$ **default terms**

$$e ::= \langle\, e_1, \ldots, e_n \;\mid\; b_{default} \;:\!-\; e_{default} \,\rangle$$

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$$\begin{array}{rcl} v & ::= & \texttt{true} \mid \texttt{false} \mid n \mid \ldots \\ & \mid & \varnothing \\ & \mid & \circledast \end{array}$$

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  ⟨ | income ≤ $10,000 :- 10%⟩,
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  | true :- 20%
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2. If **exactly 1 exception** is raised, then **return its value**
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$$\texttt{income} \leq \$10,000$$

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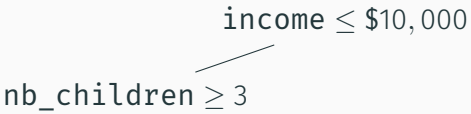

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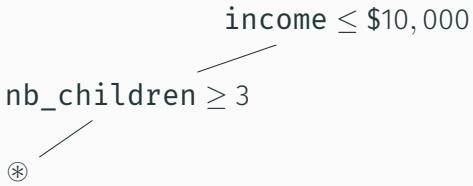

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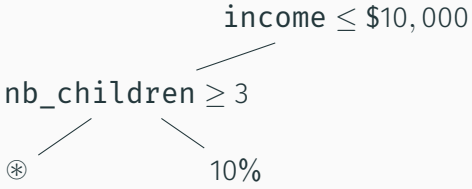

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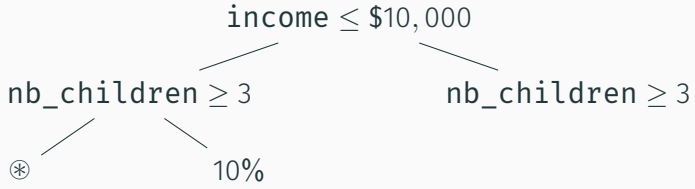
`income` = `$11,000`; `nb_children` = `4`



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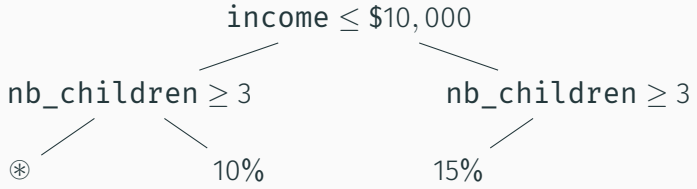

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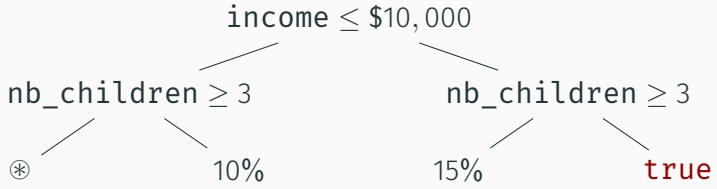

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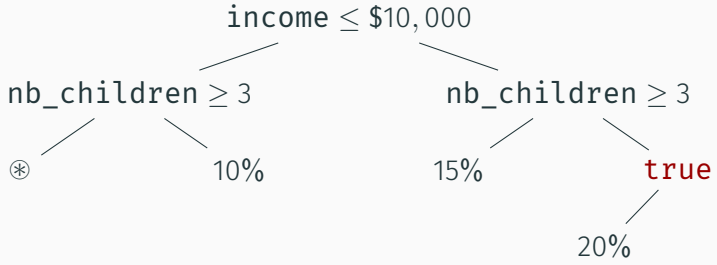

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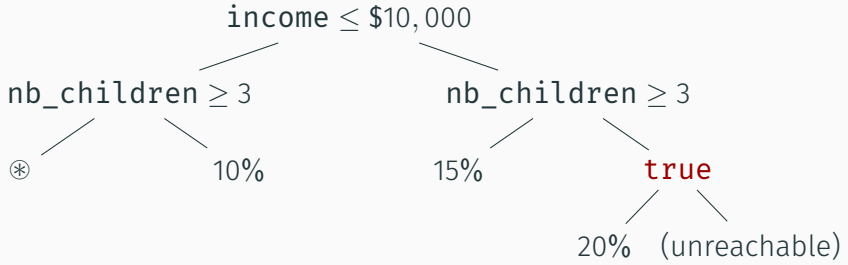
income = ???; nb_children = ???



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Why not purely symbolic execution?

- Possible incompleteness due to mixed integer/rational reasoning
- Lists are hard to encode
- Dates can be ambiguous: what is 29 February 2024 + 1 year?
- We want to generate counter-examples

# Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.

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  ⟨ | income ≤ $10,000 :- 10%⟩,
  ⟨ | nb_children ≥ 3 :- 15%⟩
  | true :- 20%
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$\rightarrow$ it becomes an exception to the exception.

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# Performance and usability improvements

# Performance optimizations using reordering

**Theorem (Independence of exception evaluation order)**
*If there is a default value $v$ such that*

$$\langle \ldots, e_i, \ldots, e_j, \ldots \mid b_{default} \ :- \ e_{default} \rangle \longrightarrow^* v,$$

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Example:

$$\langle A, B, C, \circledast \mid b_{default} \ :\!- \ e_{default} \rangle \sim \langle \circledast, A, B, C \mid b_{default} \ :\!- \ e_{default} \rangle$$

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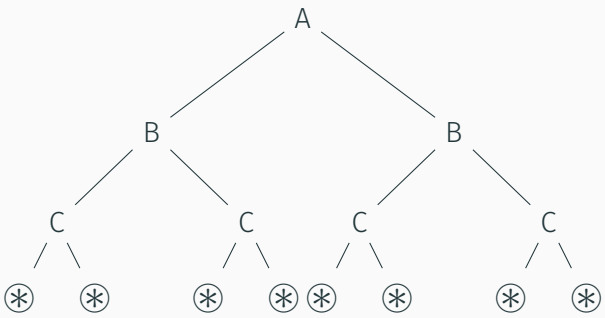


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- Difficult to compute by hand
- Find more usable input values using soft constraints
  - *e.g.* **round to $1,000**

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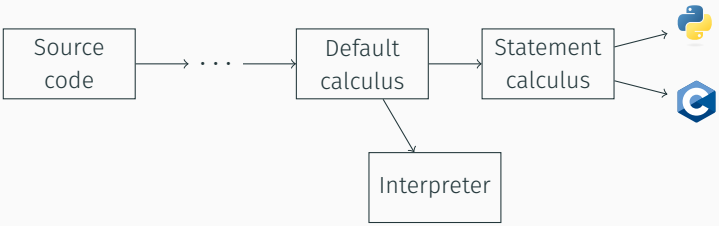
- Redundant constraint elimination

# Implementation of CUTECat



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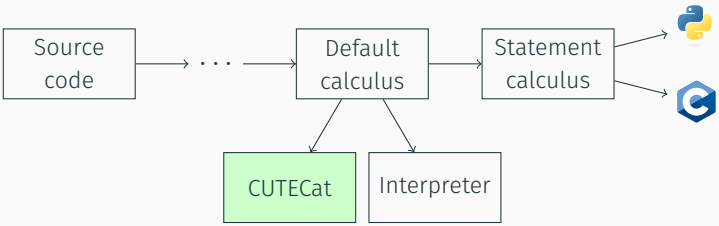


- Integrated into Catala compiler's *default calculus* IR
- 3.4k lines of OCaml code
- **Z3** SMT Solver

# Experimental evaluation

# Code base

| Law | Lines of law in Markdown | Lines of Catala | Total |
|---|---|---|---|
| French housing benefits | 5736 | 8615 | 14351 |

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| **French housing benefits** | **5736** | **8615** | **14351** |
| US Tax code § 132 | 35 | 56 | 91 |
| Minimum wage | 74 | 161 | 235 |
| Family quotient | 36 | 165 | 201 |
| Handwritten unit tests | 139 | 699 | 838 |

# Performance on small programs

| | Time (s) | | | |
|---|---|---|---|---|
| Law | No optimizations | Incremental | All opt. | Generated tests |
| US Tax code | 0.27 | 0.02 | 0.02 | 10 |
| Minimum wage | 1.01 | 0.08 | 0.08 | 17 |
| Family quotient | 82.61 | 5.21 | 4.34 | 381 |

# Case study: housing benefits

Key results

- 186,390 test cases generated in **7h of CPU time**
- 99.83% of tests satisfy soft constraints
- 366s spent in solver, the rest in evaluation
- Able to find a conflict

# Overhead of the analysis

- 4.5x overhead w.r.t. Catala interpreter
- Optimizations make SymCC[1] or SYMSAN[2] reach the same order of magnitude
- KLEE sometimes reports several orders of magnitude[3]

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- KLEE sometimes reports several orders of magnitude[3]
- Future work: more optimizations

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- Novel concolic semantics for default logic
- Integrated with Catala toolchain
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Contact, ESOP'25 preprint, slides: pierregoutagny.fr

# References i


Chen, J., Han, W., Yin, M., Zeng, H., Song, C., Lee, B., Yin, H., Shin, I.: SYMSAN: Time and space efficient concolic execution via dynamic data-flow analysis. In: USENIX Security Symposium, pp. 2531–2548, USENIX Association (2022), URL https://www.usenix.org/conference/usenixsecurity22/presentation/chen-ju

Poeplau, S., Francillon, A.: Symbolic execution with SymCC: Don't interpret, compile! In: Proceedings of the 29th USENIX Conference on Security Symposium, pp. 181–198, SEC'20, USENIX Association (2020), URL https://dl.acm.org/doi/10.5555/3489212.3489223

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# Fixing the interpretation conflict with labels

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 exception definition tax_rate
  under condition house.income <= $10,000
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```

# Article 4
For people in charge of 3 or more children, the percentage mentioned at article 1
is 15%.
```catala
scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.nb_children >= 3
  consequence equals 15%
```
```

# Fixing the interpretation conflict with labels

Suppose the lawyer says the `income` condition has priority.

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If the income is less than $10,000, the percentage mentioned at article 1 is 10%.
```catala
scope IncomeTaxComputation:
 exception children definition tax_rate
  under condition house.income <= $10,000
  consequence equals 10%
```

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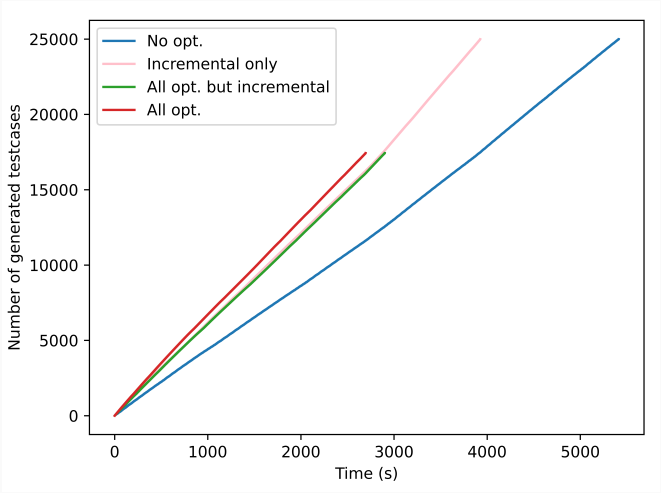
```catala
scope IncomeTaxComputation:
 label children
 exception definition tax_rate
  under condition house.nb_children >= 3
  consequence equals 15%
```
```

# Ablation study




Generated tests vs time