CUTECat

Concolic Execution for Computational Law

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Introduction

- Computational laws specify algorithms: taxes, social benefits, etc.
- Administrations implement them as programs
- Critical: *e.g.* French military payroll system Louvois: 120k military personnel over- or under-paid, overpayments totalling 545M € to pay back

The income tax is a fixed percentage of the income.

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Article 2

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If the income is less than \$10,000, the percentage mentioned at article 1 is 10%.

Article 4

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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Article 4

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Default logic

The Catala domain-specific language

```
scope IncomeTaxComputation:
    definition income_tax equals
    house.income * tax_rate
```

Article 2
The fixed percentage mentioned at
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Catala

```
scope IncomeTaxComputation:
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Literate programming

Article 3

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scope IncomeTaxComputation:
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```
scope IncomeTaxComputation:
    exception definition tax_rate
    under condition house.nb_children >= 3
    consequence equals 15%
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catala-lang.org

The Catala domain-specific language

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- Literate programming
- Follows the exception/default structure of the law

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- Ambiguities in the code
 - interpretation conflicts, e.g. income = \$9,000 and children = 4
 - unhandled cases
 - in Catala: ambiguity = runtime error
 - · resolved by lawyers/administration if implementation is correct
- Other errors: division by zero, assertion error, etc.

What properties do we want in our search for errors?

- Find errors automatically
- Systematically:
 - find complex corner cases
 - complete coverage
- Handle default logic
- Generate (counter-)examples for non-programmer users

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 \rightarrow Symbolic execution

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 - no loops or memory
 - all programs terminate

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 - Avoid some common obstacles for free:
 - no loops or memory
 - all programs terminate
 - But some features are hard to encode symbolically

Concolic execution of default terms

Performance and usability improvements

Experimental evaluation

Concolic execution of default terms

```
if x > 0
    then 0
    else if y < 10
        then y
        else error</pre>
```

Concolic = *conc*rete + symbolic

Step x y Output Constraints after evaluation Next path to try



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1 1	20	0	<i>x</i> > 0	



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$$e ::= \langle \underbrace{e_1, \ldots, e_n}_{\text{exceptions}} \mid b_{default} := e_{default} \rangle$$







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< | income ≤ $10,000 :- 10%>,
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| true :- 20%
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exception definition tax_rate
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consequence equals 15%
definition tax_rate equals 20%
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1. Evaluate all exceptions

- 2. If exactly 1 exception is raised, then return its value
- 3. Else if at least 2 exceptions are raised, then return \circledast
- 4. Else if **0** exceptions are raised, evaluate $b_{default}$ and
 - If $b_{default} =$ true, then evaluate $e_{default}$
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Default terms: semantics

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```

```
\langle | nb_children \geq 3 :- 15\% \rangle
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true:-20%

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under condition house.income <= $10,000
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exception definition tax_rate
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 $\langle \langle | \text{ income} \leq \$10,000:-10\% \rangle, \langle | \text{ nb_children} \geq 3:-15\% \rangle | \text{ true}:-20\% \rangle$

(\left(| income ≤ \$10,000 :- 10%), \left(| nb_children ≥ 3 :- 15%) | true :- 20%\left)
income = \$9,000; nb_children = 4

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(\left(| income ≤ \$10,000 :- 10%\), \left(| nb_children ≥ 3 :- 15%\) | true :- 20%\left\ income = \$9,000; nb_children = 2



(\left(| income ≤ \$10,000 :- 10%), \left(| nb_children ≥ 3 :- 15%) | true :- 20%\left)
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income = ???; nb_children = ???



Fixing the interpretation conflict

Suppose the lawyer says the **income** condition has priority.

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Performance and usability improvements

Performance optimizations using reordering

Theorem (Independence of exception evaluation order) *If there is a default value v such that*

$$\langle \dots, e_i, \dots, e_j, \dots \mid b_{default} := e_{default} \rangle \longrightarrow^* V,$$

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Example:

(A, B, C,
$$\circledast$$
 | $b_{default}$:- $e_{default}$ \sim (\circledast , A, B, C | $b_{default}$:- $e_{default}$

$$\langle A, B, C, \circledast | b_{default} := e_{default} \rangle \sim \langle \circledast, A, B, C | b_{default} := e_{default} \rangle$$

*



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Query: income > \$10,000 ?

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- Find more usable input values using soft constraints

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Query: income > \$10,000 ? → Answer: \$10,000.01 **\$11,000**

- Difficult to compute by hand
- Find more usable input values using soft constraints
 - e.g. round to \$1,000

• Incremental mode

- Incremental mode
 - Solver keeps a stack of constraints

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 - Allows efficient soft constraints
- Redundant constraint elimination

Implementation of CUTECat



Implementation of CUTECat



- Integrated into Catala compiler's default calculus IR
- 3.4k lines of OCaml code
- Z3 SMT Solver

Experimental evaluation

Law	Lines of law in Markdown Lines of Catala		Total
French housing benefits	5736	8615	14351
US Tax code § 132	35	56	91
Minimum wage	74	161	235
Family quotient	36	165	201
Handwritten unit tests	139	699	838

	Ti			
Law	No optimizations	Incremental	All opt.	Generated tests
US Tax code	0.27	0.02	0.02	10
Minimum wage	1.01	0.08	0.08	17
Family quotient	82.61	5.21	4.34	381
Key results

- 186,390 test cases generated in **7h of CPU time**
- 99.83% of tests satisfy soft constraints
- 366s spent in solver, the rest in evaluation
- Able to find a conflict

- 4.5x overhead w.r.t. Catala interpreter
- Optimizations make SymCC¹ or SYMSAN² reach the same order of magnitude
- KLEE sometimes reports several orders of magnitude³

¹Poeplau and Francillon [2020] ²Chen et al. [2022] ³Yun et al. [2018]

- 4.5x overhead w.r.t. Catala interpreter
- Optimizations make SymCC¹ or SYMSAN² reach the same order of magnitude
- KLEE sometimes reports several orders of magnitude³
- Future work: more optimizations

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- Novel concolic semantics for default logic
- Integrated with Catala toolchain
- Optimizations improve efficiency and usability by lawyers
- $\cdot \ {\sim} 200k$ test cases in less than 7h on real-world example

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- Conformance testing *e.g.* for simulator
- Improve user-friendliness for non-programmers

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Chen, J., Han, W., Yin, M., Zeng, H., Song, C., Lee, B., Yin, H., Shin, I.: SYMSAN: Time and space efficient concolic execution via dynamic data-flow analysis. In: USENIX Security Symposium, pp. 2531–2548, USENIX Association (2022), URL https://www.usenix.org/conference/usenixsecurity22/ presentation/chen-ju

Poeplau, S., Francillon, A.: Symbolic execution with SymCC: Don't interpret, compile! In: Proceedings of the 29th USENIX Conference on Security Symposium, pp. 181–198, SEC'20, USENIX Association (2020), URL https://dl.acm.org/doi/10.5555/3489212.3489223 Yun, I., Lee, S., Xu, M., Jang, Y., Kim, T.: QSYM : A practical concolic execution engine tailored for hybrid fuzzing. In: USENIX Security Symposium, pp. 745–761, USENIX Association (2018), URL https://dl.acm.org/doi/10.5555/3277203.3277260

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- Lists are hard to encode
- Dates can be ambiguous: what is 29 February 2024 + 1 year?
- We want to generate counter-examples

Suppose the lawyer says the **income** condition has priority.

```
# Article 3
If the income is less than $10,000, the percentage mentioned at article 1 is 10%.
scope IncomeTaxComputation:
 exception definition tax rate
  under condition house.income <= $10.000</pre>
 consequence equals 10%
# Article 4
For people in charge of 3 or more children, the percentage mentioned at article 1
scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.nb_children >= 3
consequence equals 15%
```

Suppose the lawyer says the **income** condition has priority.

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If the income is less than $10,000, the percentage mentioned at article 1 is 10%.
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scope IncomeTaxComputation:
exception children definition tax_rate
under condition house.income <= $10,000
.Consequence equals 10%
# Article 4
For people in charge of 3 or more children, the percentage mentioned at article 1
is 15%.
``catala
scope IncomeTaxComputation:</pre>
```

label children
exception definition tax_rate
under condition house.nb_children >= 3
consequence equals 15%

Ablation study



Generated tests vs time