

# CUTECat

## Concolic Execution for Computational Law

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# Introduction

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- **Computational laws** encode algorithms: taxes, social benefits, etc.
- Administrations implement them as programs
- Critical: *e.g.* French military payroll system Louvois: 120k military personnel over- or under-paid, overpayments totalling 545M € to pay back

## Article 1

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# Structure of computational law: income tax example

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## Article 3

If the income is less than \$10,000, the percentage mentioned at article 1 is 10%.

## Article 4

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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exception

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## Article 4

exception

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

Default logic



# The Catala domain-specific language

## # Article 1

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```
```catala
scope IncomeTaxComputation:
  definition income_tax equals
    house.income * tax_rate
  ...
```

## # Article 2

The fixed percentage mentioned at article 1 is 20%.

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- Literate programming

## # Article 3

If the income is less than \$10,000, the percentage mentioned at article 1 is 10%.

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```catala
scope IncomeTaxComputation:
  exception definition tax_rate
    under condition house.income <= $10,000
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## # Article 4

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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```catala
scope IncomeTaxComputation:
  exception definition tax_rate
    under condition house.nb_children >= 3
      consequence equals 15%
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- Literate programming
- Follows the exception/default structure of the law

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  - interpretation conflicts, e.g. `income = $9,000` and `children = 4`
  - unhandled cases

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- **Ambiguities** in the code
  - interpretation conflicts, e.g. `income = $9,000` and `children = 4`
  - unhandled cases
  - in Catala: ambiguity = runtime error
    - resolved by lawyer/court if implementation is correct
- Other errors: division by zero, assertion error, etc.

What properties do we want in our search for errors?

- Find errors automatically
- Systematically:
  - find complex corner cases
  - complete coverage
- Handle default logic
- Generate (counter-)examples for non-expert users

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- Avoid some common obstacles for free:
  - no loops or memory
  - all programs terminate

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→ Symbolic execution

- Avoid some common obstacles for free:
  - no loops or memory
  - all programs terminate
- But some features are hard to encode symbolically

Concolic execution of default terms

Performance and usability improvements

Experimental evaluation

## Concolic execution of default terms

---

## Concolic execution: first example

Concolic = *concrete* + *symbolic*

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if x > 0
  then 0
  else if y < 0
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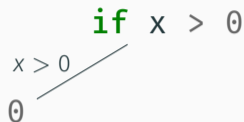
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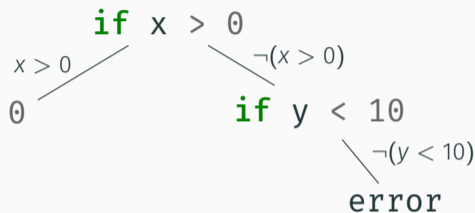
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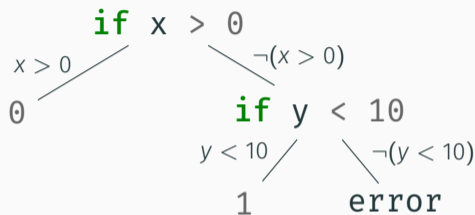


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3	0	9	1	$\neg(x > 0) \wedge y < 10$	-	

Source code  $\xrightarrow{\text{compiler}}$  **default terms**

$$e ::= \langle e_1, \dots, e_n \mid b_{\text{default}} :- e_{\text{default}} \rangle$$

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$$\quad \mid \emptyset$$
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## Default terms: semantics

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1. Evaluate all exceptions
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income = $9,000;    nb_children = 4
```

```
income ≤ $10,000
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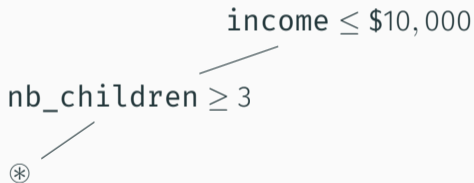
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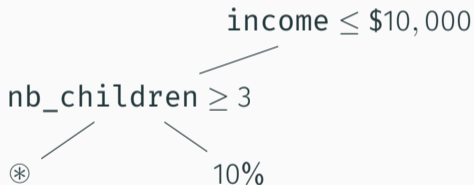
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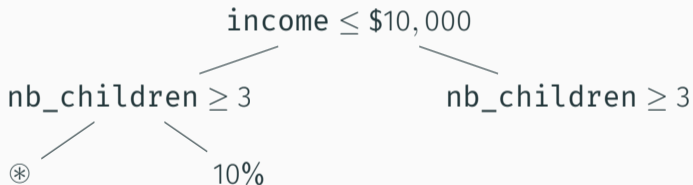
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## Concolic execution of default terms

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<< | income ≤ $10,000 :- 10%>, < | nb_children ≥ 3 :- 15%> | true :- 20%>  
income = $11,000; nb_children = 4
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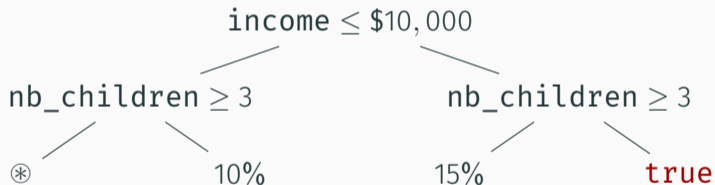
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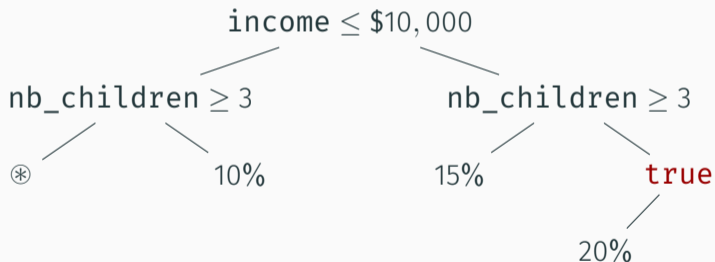
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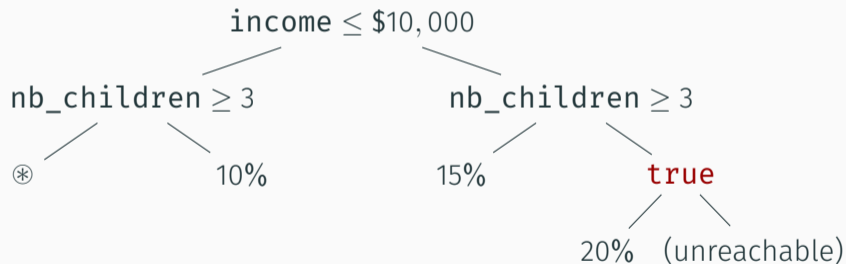
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income = ???;      nb_children = ???
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## Fixing the interpretation conflict

Suppose the lawyer says the **income** condition has priority.

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## Performance and usability improvements

---

# Performance optimizations using reordering

## Theorem (Independence of exception evaluation order)

*If there is a default value  $v$  such that*

$$\langle \dots, e_i, \dots, e_j, \dots \mid b_{\text{default}} :- e_{\text{default}} \rangle \longrightarrow^* v,$$

*then*

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Example:

$$\langle \dots, \textcircled{*} \mid b_{\text{default}} :- e_{\text{default}} \rangle \sim \langle \textcircled{*}, \dots \mid b_{\text{default}} :- e_{\text{default}} \rangle$$

## Human-compatible test cases

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- Difficult for lawyers to compute by hand
- Find more usable input values using **soft constraints**

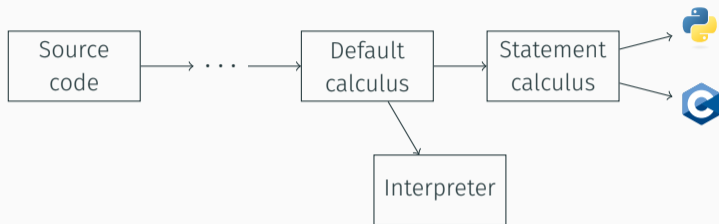
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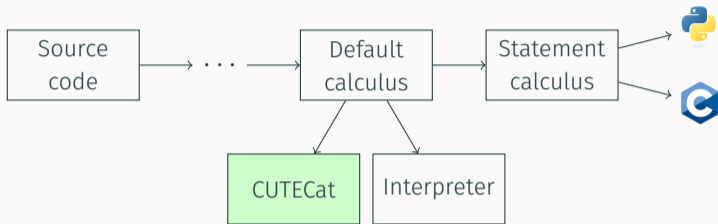
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- Difficult for lawyers to compute by hand
- Find more usable input values using soft constraints
  - *e.g.* round to \$1,000

# Implementation of CUTEcAT



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- Integrated into Catala compiler's *default calculus* IR
- 3.4k lines of OCaml code
- **Z3** SMT Solver

## Experimental evaluation

---

Law	Lines of law in Markdown	Lines of Catala	Total
French housing benefits	5736	8615	14351

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<b>French housing benefits</b>	<b>5736</b>	<b>8615</b>	<b>14351</b>
US Tax code § 132	35	56	91
Minimum wage	74	161	235
Family quotient	36	165	201
Handwritten unit tests	139	699	838



## Performance on small programs

Law	Time (s)		Generated tests
	No optimizations	Optimized	
US Tax code	0.27	0.02	10
Minimum wage	1.01	0.08	17
Family quotient	82.61	4.34	381

### Key results

- 186,390 test cases generated in **7h of CPU time**
- 99.83% of tests satisfy soft constraints
- 366s spent in solver, the rest in evaluation
- 4.5x overhead w.r.t. Catala interpreter
- Able to find a conflict

## Conclusion

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Future work:

- Complex cases *e.g.* lists and dates
- Improve user-friendliness for non-technical users

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- Optimizations improve efficiency and usability by lawyers
- **186,390 test cases** in less than **7h** on real-world example

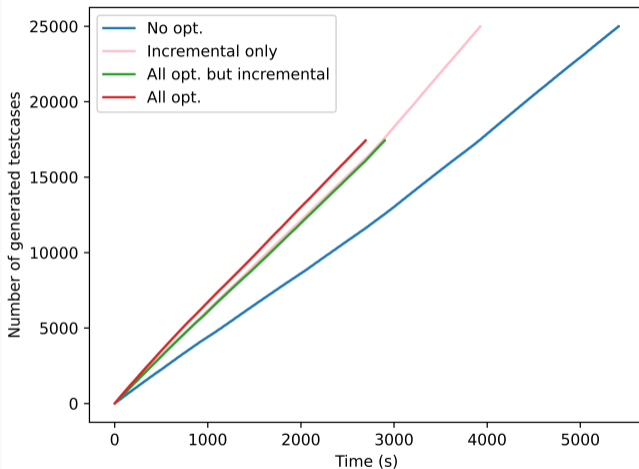
Future work:

- Complex cases *e.g.* lists and dates
- Improve user-friendliness for non-technical users

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# Ablation study



Generated tests vs time